Imperative Logic

Indicative (You’re doing A.)
A Au

Imperative (Do A.)
$\underline{A}$ $A\underline{u}$

1. Any underlined capital letter is a wff.
2. The result of writing a capital letter and then one or more small letters, one small letter of which is underlined, is a wff.

Don’t do A = $\sim\underline{A}$

Do A and B = $(\underline{A} \cdot \underline{B})$

Do A or B = $(\underline{A} \vee \underline{B})$

Don’t do either A or B = $\sim(\underline{A} \vee \underline{B})$

Don’t both do A and do B =
Don’t combine doing A with doing B = $\sim(\underline{A} \cdot \underline{B})$

Don’t combine doing A with not doing B =
Don’t do A without doing B = $\sim(\underline{A} \cdot \sim\underline{B})$

You’re doing A and you’re doing B = $(A \cdot B)$

You’re doing A, but do B = $(A \cdot \underline{B})$

Do A and B = $(\underline{A} \cdot \underline{B})$

If you’re doing A, then you’re doing B = $(A \supset B)$

If you (in fact) are doing A, then do B = $(A \supset \underline{B})$

Do A, only if you (in fact) are doing B = $(\underline{A} \supset B)$

If you (in fact) are doing A, then don’t do B = $(A \supset \sim\underline{B})$

Don’t combine doing A with doing B = $\sim(\underline{A} \cdot \underline{B})$

X, do (or be) A = $A\underline{x}$

X, do A to Y = $A\underline{x}y$

Everyone does A = $(x)Ax$

Let everyone do A = $(x)A\underline{x}$

Let everyone who (in fact) is doing A do B = $(x)(Ax \supset B\underline{x})$

Let someone who (in fact) is doing A do B = $(\exists x)(Ax \cdot B\underline{x})$

Let someone both do A and do B = $(\exists x)(A\underline{x} \cdot B\underline{x})$

Imperative Arguments

If the cocoa is about to boil,
remove it from the heat.
The cocoa is about to boil.
∴ Remove it from the heat.

$(B \supset \underline{R})$ Valid
B
$\therefore\ \underline{R}$

- Use the same inference rules as before; but treat "A" and "$\underline{A}$" as different wffs.
- An argument is VALID if it is inconsistent to join the premises with the contradictory of the conclusion.
- Alternatively, VALID = if the premises are correct ("1") then so must be the conclusion.

Don't combine accelerating
with braking.
You're accelerating.
∴ Don't brake.

*	1	$\sim(\underline{A}^0 \cdot \underline{B}^1) = 1$		Invalid
	2	$A^1 = 1$		
		$[\therefore \sim\underline{B}^1 = 0$		
	3	asm: $\underline{B}$		
	4	$\therefore \sim\underline{A}$	{from 1 and 3}	

$A, \sim\underline{A}, \underline{B}$

On our refutation:

$A = 1$
$\underline{A} = 0$
$\underline{B} = 1$

This is consistent:

You're accelerating.
Don't accelerate.
Instead, brake.

	Don’t combine accelerating with braking.		$\sim(\underline{A} \cdot \underline{B})$	Invalid
	You’re accelerating.		A	
∴	Don’t brake.	∴	$\sim\underline{B}$	
	If you’re accelerating, then don’t brake.		$(A \supset \sim\underline{B})$	Valid
	You’re accelerating.		A	
∴	Don’t brake.	∴	$\sim\underline{B}$	

$(A \supset \sim\underline{B})$	=	If you do A, then don’t believe that A is wrong.
$(B \supset \sim\underline{A})$	=	If you believe that A is wrong, then don’t do A.
$\sim(\underline{B} \cdot \underline{A})$	=	Don’t combine believing that A is wrong with doing A.

Deontic Logic

Indicative (You're doing A.)	*Imperative* (Do A.)	*Deontic* (You ought to do A.)
A	$\underline{A}$	O$\underline{A}$
Au	A$\underline{u}$	OA$\underline{u}$

3. The result of writing "O" or "R," and then an imperative wff, is a deontic wff.

$O\underline{A}$ = It’s obligatory that A.
$OA\underline{x}$ = X ought to do A.
$OA\underline{x}y$ = X ought to do A to Y.

$R\underline{A}$ = It’s permissible that A.
$RA\underline{x}$ = It’s all right for X to do A.
$RA\underline{x}y$ = It’s all right for X to do A to Y.

Act A is wrong	=	$\sim R\underline{A}$	=	Act A isn’t all right.
	=	$O\sim\underline{A}$	=	Act A ought not to be done.

It ought to be that A and B = $O(\underline{A} \cdot \underline{B})$
It's all right that A or B = $R(\underline{A} \vee \underline{B})$

If you do A, then you ought not to do B = $(A \supset O{\sim}\underline{B})$
You ought not to combine doing A with doing B = $O{\sim}(\underline{A} \cdot \underline{B})$

It's obligatory that everyone do A = $O(x)A\underline{x}$
It isn't obligatory that everyone do A = ${\sim}O(x)A\underline{x}$
It's obligatory that not everyone do A = $O{\sim}(x)A\underline{x}$
It's obligatory that everyone refrain from doing A = $O(x){\sim}A\underline{x}$

It's obligatory that someone answer the phone	=	O(∃x)A<u>x</u>
There's someone who has the obligation to answer the phone	=	(∃x)OA<u>x</u>

It's obligatory that some who kill repent	=	O(∃x)(Kx · R<u>x</u>)
It's obligatory that some kill who repent	=	O(∃x)(K<u>x</u> · Rx)
It's obligatory that some both kill and repent	=	O(∃x)(K<u>x</u> · R<u>x</u>)

Deontic Proofs

- A *world prefix* is a string of zero or more instances of "W" or "D."
- A *possible world* is a consistent and complete set of indicatives and imperatives.
- A *deontic world* is a possible world in which the indicative statements are all true and the imperatives prescribe some jointly permissible combination of actions.
- "O$\underline{A}$" is true if and only if "$\underline{A}$" is in *all* deontic worlds.
- "R$\underline{A}$" is true if and only if "$\underline{A}$" is in *some* deontic worlds.

Deontic Inference Rules

First reverse squiggles	~O<u>A</u> → R~<u>A</u> ~R<u>A</u> → O~<u>A</u>	*
and drop R's;	R<u>A</u> → D ∴ <u>A</u>, use a *new* string of D's	*
lastly, drop O's.	O<u>A</u> → D ∴ <u>A</u>, use a blank or any string of D's	Don't star

Indicative transfer

$D \therefore A \quad \rightarrow \quad A$,
the world prefixes of the derived and deriving steps must be identical except that one ends in one or more additional D's

We can transfer indicatives freely between a deontic world and whatever world it depends on.

Kant's Law

$O\underline{A} \quad \rightarrow \quad \Diamond A$

"Ought" implies "can":

"You ought to do A" entails "It's possible for you to do A."

Hare's Law:	An "ought" entails the corresponding imperative.
Kant's Law:	"Ought" implies "can."
Hume's Law:	You can't deduce an "ought" from an "is."
Poincaré's Law:	You can't deduce an imperative from an "is."

*	1	R($\underline{A}$ • $\underline{B}$)		**Valid**
		[∴ R$\underline{A}$		
*	2	┌ asm: ~R$\underline{A}$		
	3	│ ∴ O~$\underline{A}$ {from 2}	←	reverse squiggles
*	4	│ D ∴ ($\underline{A}$ • $\underline{B}$) {from 1}	←	drop "R"
	5	│ D ∴ $\underline{A}$ {from 4}		
	6	│ D ∴ $\underline{B}$ {from 4}		
	7	└ D ∴ ~$\underline{A}$ {from 3}	←	drop "O"
	8	∴ R$\underline{A}$ {from 2; 5 contradicts 7}		

1. Reverse squiggles.
2. Drop each initial "R," using a new deontic world each time.
3. Lastly, drop each initial "O" once for each old deontic world. (Never use a new deontic world when you drop "O.")

[∴ ☐(O($\underline{A}$ • $\underline{B}$) ⊃ O$\underline{A}$)   Valid

* 1 asm: ~☐(O($\underline{A}$ • $\underline{B}$) ⊃ O$\underline{A}$)

* 2 ∴ ◇~(O($\underline{A}$ • $\underline{B}$) ⊃ O$\underline{A}$)   {from 1}

* 3 W ∴ ~(O($\underline{A}$ • $\underline{B}$) ⊃ O$\underline{A}$)   {from 2}

4 W ∴ O($\underline{A}$ • $\underline{B}$)   {from 3}

* 5 W ∴ ~O$\underline{A}$ {from 3}

* 6 W ∴ R~$\underline{A}$ {from 5}

7 WD ∴ ~$\underline{A}$ {from 6}

8 WD ∴ ($\underline{A}$ • $\underline{B}$)   {from 4}

9 WD ∴ $\underline{A}$ {from 8}

10 ∴ ☐(O($\underline{A}$ • $\underline{B}$) ⊃ O$\underline{A}$)   {from 1; 7 contradicts 9}