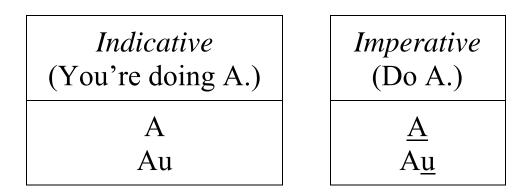
Imperative Logic



- 1. Any underlined capital letter is a wff.
- 2. The result of writing a capital letter and then one or more small letters, one small letter of which is underlined, is a wff.

Don't do A =
$$\sim \underline{A}$$

Do A and B = $(\underline{A} \cdot \underline{B})$
Do A or B = $(\underline{A} \vee \underline{B})$
Don't do either A or B = $\sim (\underline{A} \vee \underline{B})$

Don't both do A and do B = Don't combine doing A with doing B = $\sim(\underline{A} \cdot \underline{B})$

Don't combine doing A with not doing B = Don't do A without doing B = $\sim(\underline{A} \cdot \sim \underline{B})$

You're doing A and you're doing B =
$$(A \cdot B)$$

You're doing A, but do B = $(A \cdot \underline{B})$
Do A and B = $(\underline{A} \cdot \underline{B})$

If you're doing A, then you're doing B = $(A \supset B)$ If you (in fact) are doing A, then do B = $(A \supset \underline{B})$ Do A, only if you (in fact) are doing B = $(\underline{A} \supset B)$

If you (in fact) are doing A, then don't do B = $(A \supset \sim \underline{B})$ Don't combine doing A with doing B = $\sim (\underline{A} \cdot \underline{B})$

Pages 267-69

X, do (or be) A = $A\underline{x}$ X, do A to Y = $A\underline{x}y$

Everyone does A = (x)AxLet everyone do $A = (x)A\underline{x}$

Let everyone who (in fact) is doing A do B = $(x)(Ax \supset B\underline{x})$ Let someone who (in fact) is doing A do B = $(\exists x)(Ax \cdot B\underline{x})$ Let someone both do A and do B = $(\exists x)(A\underline{x} \cdot B\underline{x})$

Imperative Arguments

If the cocoa is about to boil,		$(\mathbf{D} \supset \mathbf{D})$	Valid
remove it from the heat.		$(B \supseteq \underline{R})$ B	vanu
The cocoa is about to boil.	•	D	
Remove it from the heat.	••	<u>R</u>	

- Use the same inference rules as before; but treat "A" and "<u>A</u>" as different wffs.
- An argument is VALID if it is inconsistent to join the premises with the contradictory of the conclusion.
- Alternatively, VALID = if the premises are correct ("1") then so must be the conclusion.

. .

Don't combine accelerating with braking. You're accelerating.

 \therefore Don't brake.

* 1 ~
$$(\underline{A}^{o} \cdot \underline{B}^{1}) = 1$$
 Invalid
2 $A^{1} = 1$ $A, \sim \underline{A}, \underline{B}$
[$\therefore \sim \underline{B}^{1} = 0$
3 $\operatorname{asm:} \underline{B}$
4 $\therefore \sim \underline{A}$ {from 1 and 3}

On our refutation:

$$A = 1$$
$$\underline{A} = 0$$
$$B = 1$$

This is consistent: You're accelerating. Don't accelerate. Instead, brake.

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Pages 269–73

	Don't combine accelerating with braking.		$\sim (\underline{A} \cdot \underline{B})$	Invalid
	You're accelerating.		А	
	Don't brake.		~ <u>B</u>	
	If you're accelerating, then don't brake.		$(\mathbf{A} \supset \mathbf{\sim} \underline{\mathbf{B}})$	Valid
	You're accelerating.		А	
••••	Don't brake.	·.	~ <u>B</u>	

- $(A \supset \sim \underline{B}) =$ If you do A, then don't believe that A is wrong.
- $(B \supset \sim \underline{A})$ = If you believe that A is wrong, then don't do A.
 - $\sim (\underline{B} \cdot \underline{A}) = \text{Don't combine believing that A is wrong with doing A.}$

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Pages 269–73

Deontic Logic

<i>Indicative</i>	Imperative	Deontic
(You're doing A.)	(Do A.)	(You ought to do A.)
A Au	$\begin{array}{c} \underline{A} \\ A\underline{u} \end{array}$	O <u>A</u> OA <u>u</u>

3. The result of writing "O" or "R," and then an imperative wff, is a deontic wff.

Pages 276–78

$$O\underline{A} = It's obligatory that A.$$

 $OA\underline{x} = X \text{ ought to do A.}$
 $OA\underline{x}y = X \text{ ought to do A to Y.}$

$$R\underline{A} = It's \text{ permissible that } A.$$

$$RA\underline{x} = It's \text{ all right for } X \text{ to do } A.$$

$$RA\underline{x}y = It's \text{ all right for } X \text{ to do } A \text{ to } Y.$$

Act A is wrong
$$= \sim R\underline{A} = Act A isn't all right.$$

 $= O \sim \underline{A} = Act A ought not to be done.$

Pages 276–78

It ought to be that A and B = $O(\underline{A} \cdot \underline{B})$ It's all right that A or B = $R(\underline{A} \vee \underline{B})$

- If you do A, then you ought not to do B = $(A \supset O \sim \underline{B})$ You ought not to combine doing A with doing B = $O \sim (\underline{A} \cdot \underline{B})$
 - It's obligatory that everyone do $A = O(x)A\underline{x}$
 - It isn't obligatory that everyone do A = $\sim O(x)A\underline{x}$
 - It's obligatory that not everyone do $A = O^{(x)}A\underline{x}$
- It's obligatory that everyone refrain from doing A = $O(x) \sim Ax$

It's obligatory that someone
answer the phone =
$$O(\exists x)A\underline{x}$$

There's someone who has the obligation to answer the phone $= (\exists x)OA\underline{x}$

It's obligatory that some
who kill repent =
$$O(\exists x)(Kx \cdot R\underline{x})$$

- It's obligatory that some kill who repent = $O(\exists x)(K\underline{x} \cdot Rx)$
- It's obligatory that some both kill and repent = $O(\exists x)(K\underline{x} \cdot R\underline{x})$

LogiCola L (DM & DT)

Deontic Proofs

- A *world prefix* is a string of zero or more instances of "W" or "D."
- A *possible world* is a consistent and complete set of indicatives and imperatives.
- A *deontic world* is a possible world in which the indicative statements are all true and the imperatives prescribe some jointly permissible combination of actions.
- "O<u>A</u>" is true if and only if "<u>A</u>" is in *all* deontic worlds.
- " $R\underline{A}$ " is true if and only if " \underline{A} " is in *some* deontic worlds.

Suppose that these indicatives are all true:

- I have an 8 am class.
- I ought to get up before 7 am.

D

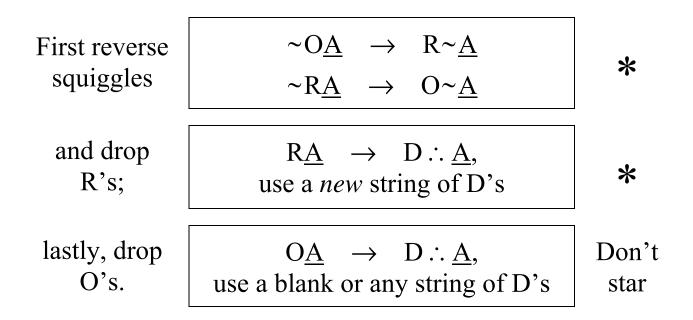
- It would be permissible for me to get up at 6:45 am.
- It would be permissible for me to get up at 6:30 am.

Then deontic worlds D and DD might contain these, in addition to the indicatives listed above:

Get up before 7 am. Get up at 6:45 am.

DD Get up before 7 am. Get up at 6:30 am.

Deontic Inference Rules



Indicative transfer

 $D:A \rightarrow A$, the world prefixes of the derived and deriving steps must be identical except that one ends in one or more additional D's

We can transfer indicatives freely between a deontic world and whatever world it depends on.

Kant's Law

$$O\underline{A} \rightarrow \Diamond A$$

"Ought" implies "can": "You ought to do A" entails "It's possible for you to do A."

Hare's Law: An "ought" entails the corresponding imperative. Kant's Law: "Ought" implies "can."

Hume's Law: You can't deduce an "ought" from an "is."

Poincaré's Law: You can't deduce an imperative from an "is."

* 1
$$R(\underline{A} \cdot \underline{B})$$
 Valid
[$\therefore R\underline{A}$
* 2 asm: $\sim R\underline{A}$
3 $\therefore O \sim \underline{A}$ {from 2} \leftarrow reverse squiggles
* 4 $D \therefore (\underline{A} \cdot \underline{B})$ {from 1} \leftarrow drop "R"
5 $D \therefore \underline{A}$ {from 4}
6 $D \therefore \underline{B}$ {from 4}
7 $\Box D \therefore \sim \underline{A}$ {from 3} \leftarrow drop "O"
8 $\therefore R\underline{A}$ {from 2; 5 contradicts 7}

- 1. Reverse squiggles.
- 2. Drop each initial "R," using a new deontic world each time.
- 3. Lastly, drop each initial "O" once for each old deontic world. (Never use a new deontic world when you drop "O.")

$$\begin{bmatrix} : : \Box(O(\underline{A} \cdot \underline{B}) \supset O\underline{A}) & Valid \\ * 1 & asm: \sim \Box(O(\underline{A} \cdot \underline{B}) \supset O\underline{A}) \\ * 2 & : \diamond \sim (O(\underline{A} \cdot \underline{B}) \supset O\underline{A}) \quad \{from 1\} \\ * 3 & W : : \sim (O(\underline{A} \cdot \underline{B}) \supset O\underline{A}) \quad \{from 2\} \\ 4 & W : : O(\underline{A} \cdot \underline{B}) \quad \{from 3\} \\ * 5 & W : : \sim O\underline{A} \quad \{from 3\} \\ * 6 & W : : R \sim \underline{A} \quad \{from 5\} \\ 7 & WD : : \sim \underline{A} \quad \{from 6\} \\ 8 & WD : : (\underline{A} \cdot \underline{B}) \quad \{from 4\} \\ 9 & WD : : \underline{A} \quad \{from 8\} \\ 10 \quad : : \Box(O(\underline{A} \cdot \underline{B}) \supset O\underline{A}) \quad \{from 1; 7 \text{ contradicts 9}\} \\ \end{bmatrix}$$